

An Ipsative Clustering Model for Analyzing Attitudinal Data

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This paper defines a model for analyzing the structure of attitude data, identifies valid methods for identifying distances (resemblances) when estimating groups of similar people, and shows in practical and theoretical terms when and why the model should be used. The model allows for respondents being in a particular social aggregate and for two individual (i.e., ipsative) effects: (1) of mean response level (an individual responding high or low compared to the group); and (2) of amplitude (narrow to wide response profile compared to the group). The appropriate resemblance measure for this model is based on the Pearson correlation, r_p , calculated between objects (e.g., people). Three alternative transformations of r_p were examined: $1 - r_p$, $\arccos(r_p)$, and the cord of angle distance. The best distance measure for the model is $\arccos(r_p)$ or $\arccos(r_p)^2$, although the cord produces similar results. Simulation results show how some resemblance coefficients (e.g., Euclidean distance) can be inappropriate and yield invalid clusters. In using r_p it is important to consider bimodality in ipsative factors because r_p cannot detect clusters that collapse on each other under ipsative transformation. Finally, it is noted that for some types of attitudinal data (e.g., performance variables with an absolute zero point), alternative resemblance measures (e.g., cosine) should be considered.

KEYWORDS: *Cluster analysis, ipsative measures, resemblance coefficients, attitude scales, response profiles.*

Patterns of behavior depend in part on patterns of attitudes and beliefs (Burt, 1937; Fishbein & Ajzen, 1975). Individuals in a given social aggregate are expected to report consistent patterns of behavior and attitudes, while the response sets for people in different social aggregates may vary substantially (Ditton, Goodale, & Johnsen, 1975; Driver & Knopf, 1977; Jackson, 1989). When inhomogeneous populations are not recognized and accounted for, the use of certain analysis techniques (e.g., factor analysis) is questionable, can render research conclusions invalid, and can lead to erroneous management actions (Beaman, 1975; Beaman & Lindsay, 1975; Cunningham, Cunningham, & Green, 1977; Guilford, 1952; Greenleaf, 1992; Bucklin & Gupta, 1992). If the appropriate model of reality identifies different social aggregates with different wants and needs, an analysis strategy is needed to correctly identify members of the groups and their associated attributes. Only

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when such groups are validly recognized and analyzed properly can management policies correctly address the demands of competing groups.

This paper is concerned with finding aggregates of people who have similar attitude profiles, and are thus postulated to display similar behaviors. We define a model for the structure of attitude data for people who are in different groups or clusters which allows for (1) individual, ipsative effects (a respondent having a personal average or modal score across a number of variables), and for (2) ipsative amplitude effects of narrow to wide response patterns (standard deviation around a mean or modal deviation). Based on the model, it is argued that the Pearson Correlation (r_p) between objects (e.g., people) provides the foundation for a valid measure of resemblance between individuals based on their attributes. Three transformations of r_p are examined as distance measures for cluster analysis to derive groups of similar people. Of the three, the arccos (r_p) or arccos (r_p)² is recommended.

An Overview of Cluster Analysis

Cluster analysis is a general set of methodological tools for estimating groups of similar objects. Similarity is usually based on resemblance coefficients derived from an object's attributes (Romesburg, 1979, 1990). Applications of cluster analysis to recreation have evaluated people (objects) on attributes such as participation rates (Romsa, 1973; Ditton et al., 1975), or motivations for engaging in an activity (Hautaloma & Brown, 1978; Manfredo & Larson, 1993). In other recreation studies (Dawson, Hinz, & Gordon, 1974; Romesburg, 1979), the objects were elements of the physical environment (e.g., sites along a hiking trail or river), while the attributes were characteristics of these physical objects (e.g., the presence or absence of different plant species, the extent of human impact).

Regardless of the choice of object and attributes, cluster analysis typically includes five steps (Romesburg, 1990). Step 1 involves constructing a data matrix. In common computer programs (e.g., SAS, SPSS), the rows in the matrix represent objects (e.g., individuals), while the columns are attributes (e.g., participation rates or responses on attitude variables). By convention, cluster analyses exploring resemblance among objects are called Q-techniques; procedures examining relationships among attributes are called R-techniques (Aldenderfer & Blashfield, 1984; Sneath & Sokal, 1973). This paper focuses on Q-techniques. Step 2 involves transforming the data (e.g., by recoding the attributes' units of measurement into dimensionless units). Although this step is optional, different standardization procedures can have dramatic consequences. In step 3, a coefficient measuring the resemblance as a similarity, or dissimilarity, distance between each pair of objects is calculated, resulting in a resemblance matrix. A variety of resemblance coefficients are available; for example, Euclidean distance or the Pearson product moment correlation, r_p (Romesburg, 1990, Chapters 8 and 10). Step 4 involves selecting a clustering method (e.g., UMPGA - Unweighted pair-group method using arithmetic averages) that may result in a tree giving the esti-

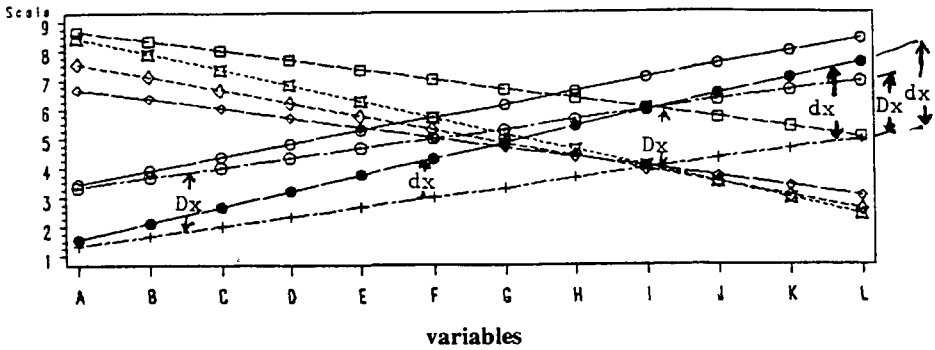
mated resemblance among objects from which clusters are identified (Aldenderfer & Blashfield, 1984, Chapter 3; Romesburg, 1990, Chapter 9). A cluster is a set of one or more similar objects. At one extreme, each object (person) is considered a separate cluster; at the other extreme, all objects are grouped into a single cluster. Step 5 examines the goodness of fit of the resemblance coefficients to the estimated clusters. Some sources recommend using the cophenetic correlation coefficient (Romesburg, 1990), while others recommend other tests (Aldenderfer & Blashfield, 1984).

Defining Resemblance: An Example to Aid Intuition

Asking individuals to respond to attitude scales is different than asking about frequency of participation (Cattell, 1944). Participation questions involve using unambiguous numbers that have common meaning to all individuals (e.g., I did not engage in the activity [0 participation] or I participated 3 times). Responses to attitudinal items are influenced by the number of scale points (Cox, 1980), the inclusion or omission of a neutral point (Dawes & Smith, 1985; Gilljam & Granberg, 1993), the choice of scale labels (Krosnick & Berent, 1990), and the numeric values assigned to a rating scale labels (Schwarz, Knauper, Hippler, Noelle-Neumann, & Clark, 1991). In addition, some individuals rate higher or lower on average, while others show wider variability in their scoring (Brown & Daniel, 1990; Greenleaf, 1992; Hui & Triandis, 1985). These latter differences are the focus of this paper. Such differences may be attributed to ipsative (personal) effects (Cattell, 1944) or may be influenced by the social group and the society to which the individual belongs (Cunningham, et al., 1977).

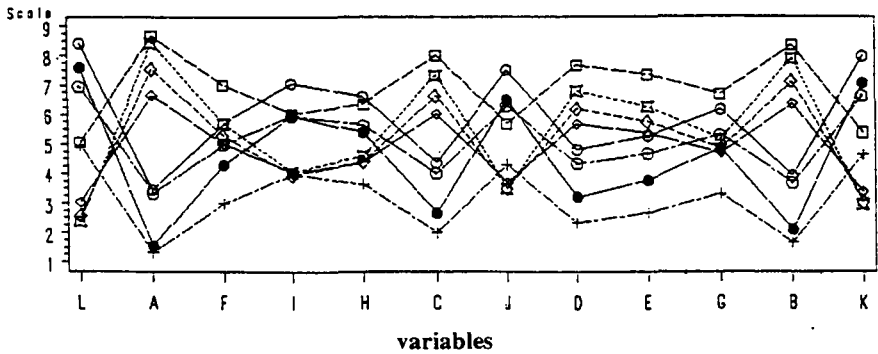
When answering a number of questions, an individual's response pattern or profile is defined. Questions arise regarding how to analyze these response patterns (Burt, 1937; Cattell, 1944, 1949; Cattell, Balcar, Horn, & Nesselrode, 1969). Figure 1A shows hypothetical data for eight people to illustrate some problems. For individuals 1 to 4 participation decreases (left to right), while for 5 to 8 there is an increase. *If the Y-axis values are frequencies of participation*, the individuals represented by the curves are all seen to differ. All eight "individuals" are statistically unique because the straight lines of Figure 1A would not occur by chance. However, *if the Y-axis in Figure 1A is attitude scores* and the goal is to identify people with common rating patterns, different interpretations of the data are possible. For example, similarity based on the "shape of a curve" might imply that persons 1 through 4 belong together because they have negative slope patterns. Persons 5 through 8 might belong together because they have a positive slope line. Figure 1A, however, may represent three or four groups rather than two. Persons 2 and 4 might be expressing their concerns with activities against the same relative pattern of importance as persons 1 and 3 are expressing their support of activities.

Assume the curves in Figure 1 are responses to attitude items. Although persons 1 and 2 generally rate higher than persons 3 and 4, these curves can be made closer by performing a type of ipsative transformation (i.e.,



Dx - shows a mean level difference that is constant between two objects or persons
 dx - shows a proportional difference between two individuals about their mean

Figure 1.A Curves showing obvious pattern relationships among variables



Social Aggregate	1	2	3	4
	5	6	7	8

This legend applies to both Figure 1.A and 1.B

Figure 1.B Curves showing no obvious pattern relationships among variables

subtracting out an individual's high or low scoring tendency as expressed by the individual's mean, mode or other value). Romesburg (1990, p. 94) calls this an additive translation. For persons 1 and 2 who rate with a wider swing than persons 3 and 4, multiplying values by a positive scale factor after subtracting out individual means changes *amplitude* so that any negative slope curve can be made close to another one. The amplitude transformation is another ipsative transformation that forces variation around means to a common unit for comparing data (Hicks, 1970; Jackson & Alwin, 1980). Using a

negative multiplier to change a positive slope pattern to a negative slope results in positive slope curves defined as similar to negative slope ones, but such reversing or related mirror image transformations (Cohen, 1969) are not allowed here. In our analyses, when two patterns are similar but one pattern is the shifted negative image of the other, it is assumed that maximum dissimilarity has been reached.

To recognize similarities between individuals' attitudes, one must determine the level of similarity between attitude patterns. An individual's mean and amplitude should not be treated as irrelevant in examining these response patterns (Cattell, 1944; Hicks, 1970; Hui & Triandis, 1985). The danger is that *individual differences in mean or amplitude on response scales* can be deceptive. Such factors can be confused with variance that should *not* be measured (Cattell, 1949; Romesburg, 1990, Chapter 8). For example, if curves 1 and 2 are considered similar, yet simply separated by 2 units, a similarity measure such as the sum of squared differences between the two curves should not count the D_x 's shown (e.g., see Cronbach & Glesser, 1953). Squaring differences to compare profiles should be taken after a raw score ipsative transformation of both curves to a mean (e.g., of zero) has been applied. A second raw score transformation is also needed to change all amplitudes to unit amplitudes. The d_x 's shown in Figure 1A illustrate amplitude related distances between curves that should not be in a sum of squares measuring pattern difference. Because ipsative factors carry information (Greenleaf, 1992), they cannot be ignored.

Contrary to what one might infer from Figure 1A, finding a pattern is not trivial. Figure 1B shows the same data as Figure 1A except that the order of the variables is changed. No patterns are obvious. Because pattern recognition should not depend on variable order, a valid analysis technique must recognize the patterns that are *not* seen in Figure 1B as clearly as the patterns seen in Figure 1A.

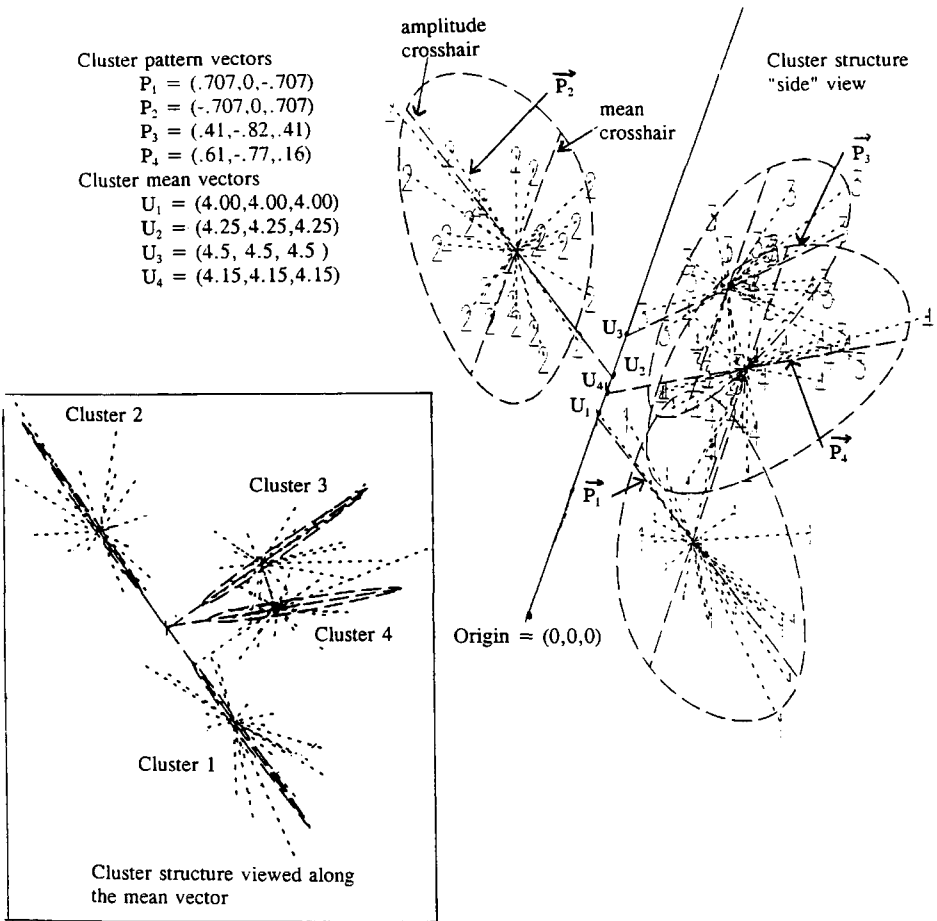
A Formal Introduction of Models

Wilderness users, campers, people using a city park, or beach visitors are not homogeneous groups. Subgroups can be distinguished by different attitudes. One may wish to develop attitude scales for people in different social aggregates based on a variety of attributes (Brown & Daniel, 1990; Schroeder, 1984; Greenleaf, 1992). To understand *if unique social aggregates* underlie a scale, a model of how people in subgroups respond to attitude questions is necessary (Zubin, 1936; Burt, 1937; Cattell, 1949; Cattell, et al., 1969; Cohen, 1969).

Figure 2 provides an intuitive basis for a formal model.¹ Four "general" mean values, U_A , around which *on average* an aggregate's members respond

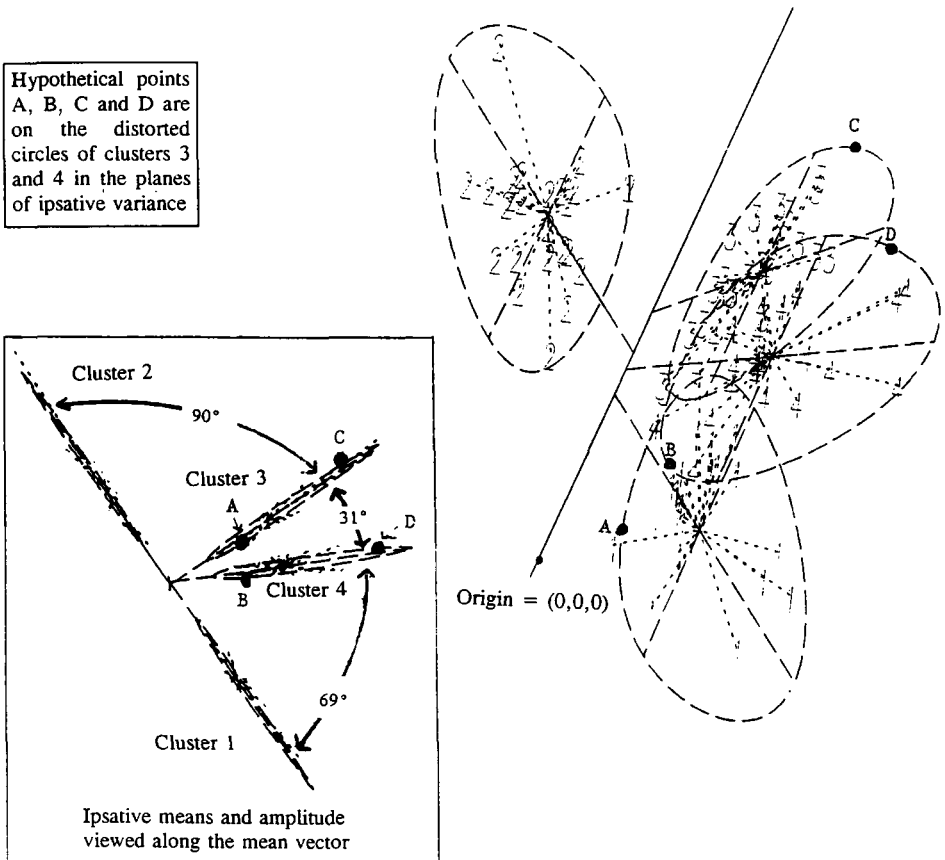
¹Three dimensional vectors are used because 3-D vectors and related graphics are readily understood. Grenier and Beaman (1993) present a similar example for 24 variables (24 dimensional vectors) and 5 clusters.

on attitude questions are shown. U_A is a vector [e.g., ($U_1 = 4, 4, 4$) ($U_2 = 4.15, 4.15, 4.15$) ($U_3 = 4.25, 4.25, 4.25$) or ($U_4 = 4.5, 4.5, 4.5$)]. Associated with the different U_A vectors, are P_A , pattern vectors, for 4 different aggregates (Figure 2). The four P_A vectors, together with their mean vectors, represent 4 hypothetical social aggregates that do not have the same attitudes. The data points in Figure 2 (labeled by 1, 2, 3, 4) represent people in four clusters. Twenty five people are shown in each cluster. Each point is defined by adding a group mean value, a pattern value ($C_A P_A$), and variable-by-vari-



able response variability (random "error", $e_{A,i}$). Data in Figure 2 do *not* reflect ipsative mean or amplitude factors. In contrast, data points in Figure 3 were created with ipsative factors being simulated without variable-by-variable random error.

Two individual (ipsative) aspects of attitude responses can be noted (Figure 3). The first concerns an individual's mean level of high / low rating. The second is amplitude which refers to individual narrow / wide swing patterns. For people in a group with similar patterns of attitudes, some generally respond high on scales, others low, and the rest between. An individual, i , may generally rate below the group's general mean of 4 (e.g., rate with a personal factor $u_{A,i} = -1$), while another rater, j , in the group rates above the group



For an enhanced 3D effect, see Figure 4.

Figure 3 Ipsative means and amplitudes without variable by variable random variance

average. The high rater with $u_{A,j} = 1$ has an overall mean $4 + 1 = 5$. In addition to high and low raters, "extreme raters" and "conservative raters" can be identified. A general pattern, P_A , with an average (or modal) amplitude C_A gives $C_A P_A$. This is a "typical" or "average" pattern for a group. A conservative rater might have a pattern amplitude of $C_A + c_A$ (e.g., $2 + [-1] = 1$), while an extreme rater may have an amplitude factor of $2 + 1 = 3$. Both share the pattern P_A , but as described for Figure 1, the shared pattern is more easily recognized with a common amplitude of 2 rather than with two different amplitudes.

The preceding provides the rationale for Equation 1. For individual, i , in aggregate A , individual responses on attitude questions allow for individual (ipsative) scale variability, $u_{A,i}$, as well as for a generally high/low rater, and an individual (ipsative) scale amplitude variability, $c_{A,i}$. The error term, $e_{A,i}$, for random variable-by-variable error is treated as additive since scales are generally bounded above and below. We believe that a term like $(C_A + c_{A,i})$ with error $(P_A + e_{A,i})$ showing higher variable-by-variable "error" for persons who have wide swings (large $C_A + c_{A,i}$) simply cannot be a good approximation of reality, where scales typically have quite narrow bounds (e.g., 5, 7, or 9 point scales).

$$\text{Equation 1: } X_{A,i} = (U_A + u_{A,i}) + (C_A + c_{A,i}) P_A + e_{A,i}$$

where:

- $X_{A,i}$ is a vector of scores, $[x(1,i), x(2,i), \dots, x(N,i)]$ where A is the social aggregate and i is the individual responding for variables, v , with $v = 1, 2, \dots, N$.
- U_A is a vector defining the general mean level for a pattern of scores for a social aggregate (e.g. see lines 1 of table 1 $[U(A), U(A), \dots, U(A)] = [4, 4, \dots, 4]$)
- $u_{i,A}$ is a vector that defines an individual i 's average displacement from U_A (e.g. $[-0.7, -0.7, \dots, -0.7]$).
- C_A is the "usual" amplitude of a pattern P_A , for a social aggregate A .
- $c_{i,A}$ is a constant that reflects an individual's amplitude in relation to C_A (broader, $c_{i,A} > 0$, or narrower, if $0 > c_{i,A} > -1$).
- P_A is a vector $[P_A(1), P_A(2), \dots, P_A(N)]$ for an aggregate, A defining an average pattern, centered around U_A , by being orthogonal to it ($yP_A(i) = 0$).
- $e_{A,i}$ specifies random variables that defines the variable by variable variance associated with each pattern variable, $[P(1), P(2), \dots, P(N)]$.

Understanding Resemblance Estimation Problems Associated with Cluster Membership

Analyses that refer to the existence of differing groups distinguishable by differing attitudes implicitly assume Equation 1 or some similar equation. In many studies, however, after making it clear that there are differing

groups, researchers compute overall average ratings, and search for patterns using multivariate techniques such as R-mode factor analysis. Because of the mathematical structure of data based on Equation 1, cluster analysis should occur prior to application of other multivariate techniques (Everitt, 1979, p. 170). Factor analysis and *overall means* that ignore aggregates are generally inappropriate (Guilford, 1952; Cattell, et al., 1969). Although the model based on Equation 1 is only a reasonable approximation for some data, it is an appropriate starting point for analyzing many attitudinal data sets currently collected in leisure research.

For interval scale data (e.g., frequency of participation), cluster analysis based on Equation 2 is appropriate (Beaman & Lindsay, 1975):

$$\text{Equation 2: } X_{A,i} = U_A + P_A + e_{A,i}$$

Equation 2 implies a model with variable-by-variable variation about a pattern based on data where scale and origin do not depend on the individual. Many cluster analyses implicitly treat data as interval scale without individual effects, and define resemblance by Euclidean distance (Romesburg, 1990, Chapters 8 and 10). Because of individual (ipsative) effects, the resemblance measures that may be appropriate for frequency of participation in activities are not necessarily appropriate for attitude data conforming to Equation 1. Even for participation data, however, one must consider that the frequency may have an $e_{A,i}$ that is highly skewed meaning that certain clustering approaches that tend to find symmetric clusters will not produce valid results. Alternative models are needed for a variety of different types of attitude data. Methodologically, our treatment of attitude data is consistent with research findings on scale properties (Havlicek & Peterson, 1977; O'Brien, 1979) and theories on level of measurement (Borgatta & Borhstedt, 1980).

Understanding the resemblance coefficient estimation problems when data conform to Equation 1, involves defining resemblance appropriately. As with all cluster analysis, the goal is to place similar things together and to put dissimilar things in different clusters (Everitt, 1980; Aldenderfer & Blashfield, 1984). If two people are in the same cluster, the distance between them should be a positive number (e.g., near zero) that reflects only residual random error. This residual error is the random differences between "within group" data. If people are in different clusters, inter-cluster distance combines with random error and should yield distances that are *on the average* larger than within cluster distances. Distance in clustering is frequently related to a sums of squares of differences (Equation 3), called Euclidean distance. SAS, for example, only supports computing distances this way (either with or without variable standardization).

$$\text{Equation 3: } D(i,j)^2 = \text{Sum Over Variables } ((x(v,i) - x(v,j))^2)$$

To visualize the estimation problem raised by a model based on Equation 1, it is useful to examine how data that conform to the model typically

look in space. In Figures 2 and 3, all general means lie along the *mean line* going through the middle of the "first quadrant." As indicated, P_1 , P_2 , etc. are average pattern vectors that are at a right angle to the end of their mean vectors. In both figures, each pattern vector defines part of a "crosshair." The data for a cluster are found "around" the crossing of crosshairs. Data with no random error and no individual mean or amplitude factors are only composed of a mean component and a pattern component; such observations fall exactly at the crossing of the crosshairs. In the figures, the crosshairs are positioned inside "distorted circles" that define the general areas in which *most* simulated observations lie.

Introducing variability as individual deviations from group means, $u_{A,i}$, results in movement along the direction of the general mean; a crosshair line parallel to the mean vector. In other words, changing the mean by an ipsative factor causes the pattern vector to be repositioned on a crosshair line up or down along the mean vector. The pattern vector here is by definition orthogonal to the mean. Ipsative variation in amplitude, $c_{A,i}$, results in movement along the amplitude crosshair line defined by the pattern vector. This is in a direction perpendicular to the general mean, and thus perpendicular to the ipsative mean line that is parallel to the general mean vector. The data points in the Figure 3 present the results of simulating combined ipsative amplitude and mean variation. The data illustrate how combining *individual* mean and amplitude variation causes spreading of data within clusters *even when there is no random variable-by-variable variability*. This spreading or individual (ipsative) variability is *within cluster* variance. The distorted circle patterns of data for clusters with the variability shown in Figure 3 illustrate that individual amplitude can go up to a large value, but only down to zero. This effect was achieved by simulating variation by triangular distributions, and skewing the distributions using exponential transformations.

Data for the simulated aggregates are most dense near the crosshairs, but since ipsative mean or amplitude variability could also be bimodal, high density could appear in a variety of patterns. The paddle shaped "pattern" of points of Figure 3 result from movement in *two dimensions*, amplitude and mean, which produces *a plane*. For each cluster, all ipsative scattering for each pattern, P_A , *only lies within a plane* (illustrated in Figure 3 by the view "almost" down the mean vector). There is no ipsative variability outside the ipsative planes of clusters. Variability outside these planes only occurs when variable-by-variable variation (i.e., random error) is introduced. The appearance of random error *without* ipsative variation is seen in Figure 2 by the view down the mean vector.

Distinguishing random variable-by-variable error from ipsative error lies at the heart of the problem of establishing who is in which cluster. Figure 2 shows random variable-by-variable variation without ipsative variability, while Figure 3 presents only ipsative variability in individual means and amplitudes. The balls of points in Figure 2 depict clusters as they are typically visualized when Euclidean distance is used in cluster analysis. All variability in these

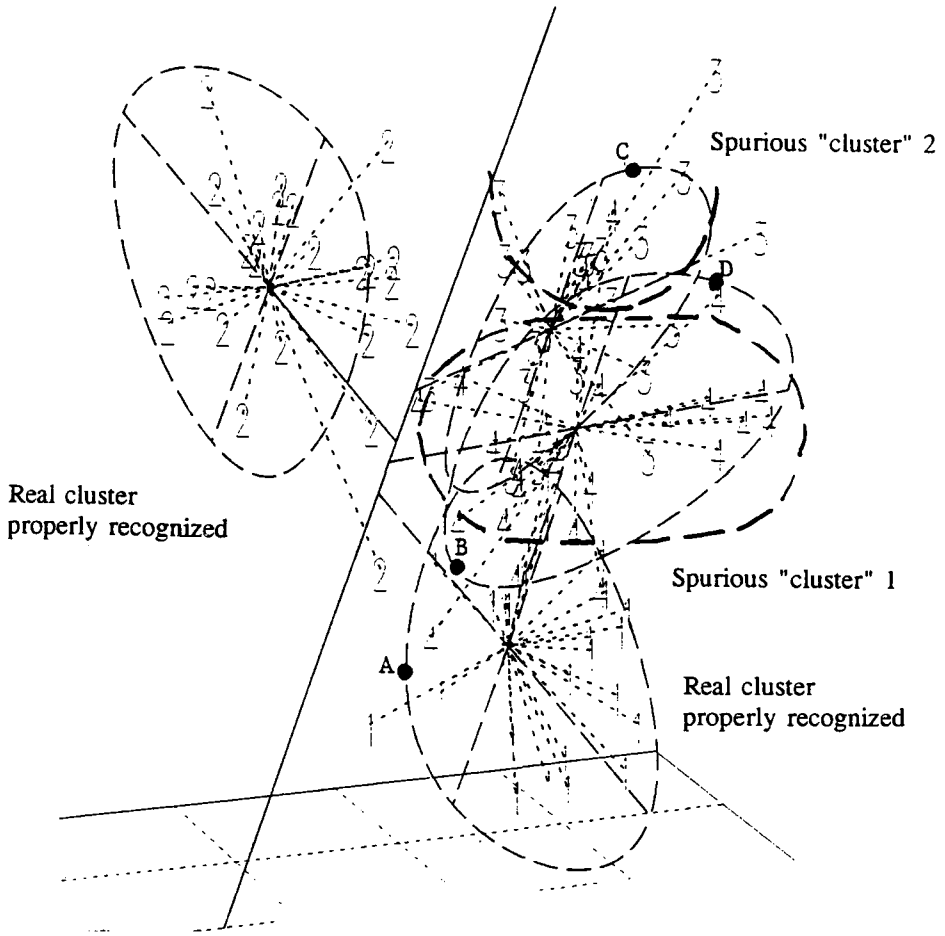
clusters is random and occurs around the intersection of crosshairs. The balls are the result of simulating data using only the terms of Equation 1 that are also in Equation 2. For this type of data, the Euclidean distance resemblance measure is appropriate and is better than several alternatives (Romesburg, 1990, Chapter 9).

The level of simulated random variable-by-variable error shown in the figures is large enough to produce variation around each crosshair, yet some separation between the "balls" of different aggregates is apparent. The variability used amounts to a reasonably high probability that a person will respond above or below their clusters average response. Equation 1 implies that the variable-by-variable balls seen around the crosshairs in Figure 2 can appear at all possible "individual points" of Figure 3. The variabilities of Figures 2 and 3 are combined in Figure 4 which shows 25 randomly selected combinations of individual amplitude ($c_{A,i}$) and mean ($u_{A,i}$) factors for each cluster. In Figure 4, the "balls" of Figure 2 spread over the planes of Figure 3 resulting in long flat blobs that resemble thick tennis racket heads.

The preceding addresses why data that are clearly clustered in Figure 2 are not clearly clustered in Figure 4. If the dissimilarity between individuals is computed using a traditional Euclidean distance as in Equation 3, one can argue that incorrect results are obtained. The observable closeness of points A and B, and of C and D in the Figure 4 clearly applies to a large number of cluster members. Points A and B, and points C and D are close, but A and D are in cluster 4, while B and C are in cluster 3. Using Euclidean distance (Equation 3) to cluster the raw data may result in improper assignment of observations to clusters because the distances between objects does not correspond with cluster membership.

The ipsative variation in individual means and amplitudes resulting in *within cluster variation* does not exist for data that conform to Equation 2. Individual or ipsative factors for that model are implied to be zero. For Equation 1, however, as the A, B, C and D illustrate, individuals in the same cluster may have data that extend over an ill-defined range. Using algorithms for finding clusters that look for hyperspheres, these blobs will not be properly found. This estimation problem does not arise in clustering where Equation 2 applies for equal $e_{A,i}$ and one has spheroids or ellipsoids if $e_{A,i}$ are not all equal (e.g., see SAS, 1988, Chapter 6).

Distances computed using raw data shown in Figures 1, 3, and 4 are expected to measure ipsative differences between individuals as well as pattern differences (Cattell, 1949; Cronbach & Gleser, 1953). This drastically distorts or biases the "real" or theoretically correct distance pattern for situations like those illustrated. There are, however, alternatives to distance computed by comparing raw data using sum of squared differences as specified in Equation 3. These involve both standardization and the use of a variety of resemblance coefficients (Romesburg, 1990, Chapters 7, 8 and 10) to compensate for individual (ipsative) variability. The solution to correctly measuring similarity or distance for people who are in the same group based



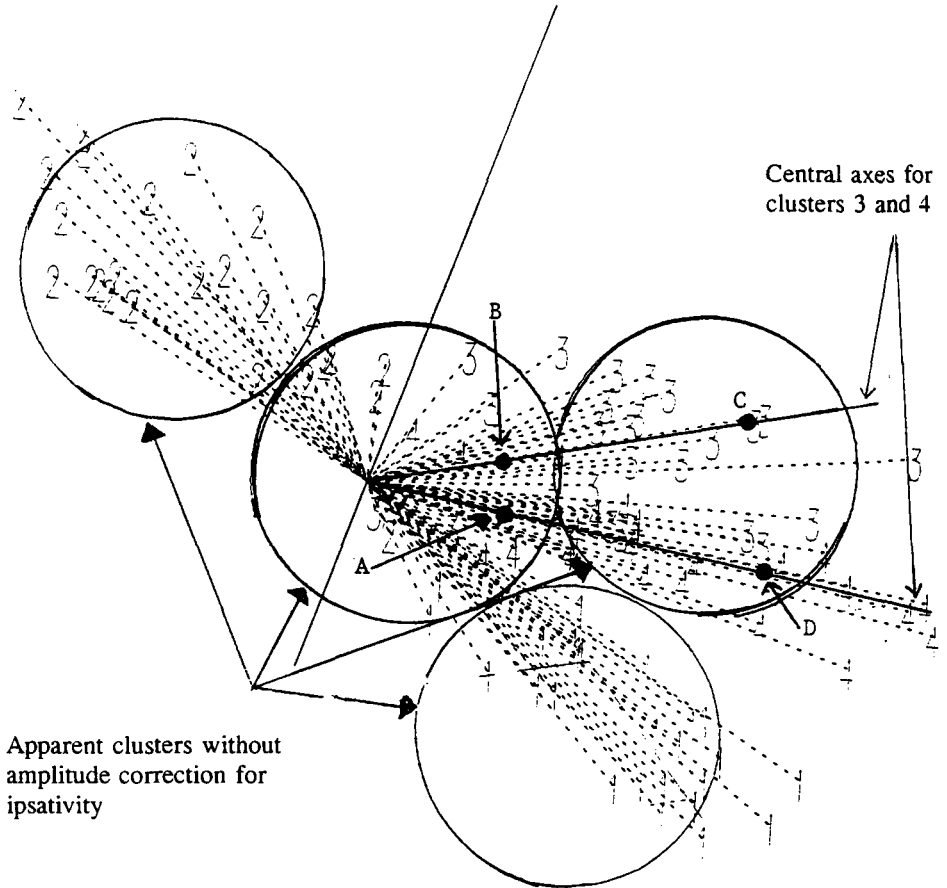
This figure illustrates what real data would look like when the random variable by variable variance of Figure 2 is combined with the ipsative scattering of individuals shown in Figure 3. The polygons differentiate "real clusters" from "spurious clusters". The points A, B, C, and D correspond to the points in Figure 3.

Figure 4 Ipsative scattering with variable by variable random variance

on Equation 1 lies in reducing the variation associated with (1) individual high/low ratings and (2) individual variation in amplitude.

Individual high/low ratings can be compensated for by removing an individual mean, mode, or some such measure. Let IM_i be an individual

ipsative mean correction for an individual, i . The ipsative transformation is $x_m(i,v) = (x(i,v) - IM_i)$. After eliminating individual means, one can compare $x_m(i,v)$ values for two individuals using Equation 3. For the Equation 1 model, however, there is still the ipsative amplitude variability between individuals to be considered. Figure 5 shows the results of the transformation to $x_m(i,v)$. The distorted circles become more narrow and all have the same origin. Because clusters remain elongated, using Equation 3 with $x_m(i,v)$



This figure illustrates what the real data from Figure 4 would look like when ipsative centering is corrected. The points A, B, C, and D correspond to the points in Figure 3.

Figure 5 Correcting for ipsative means

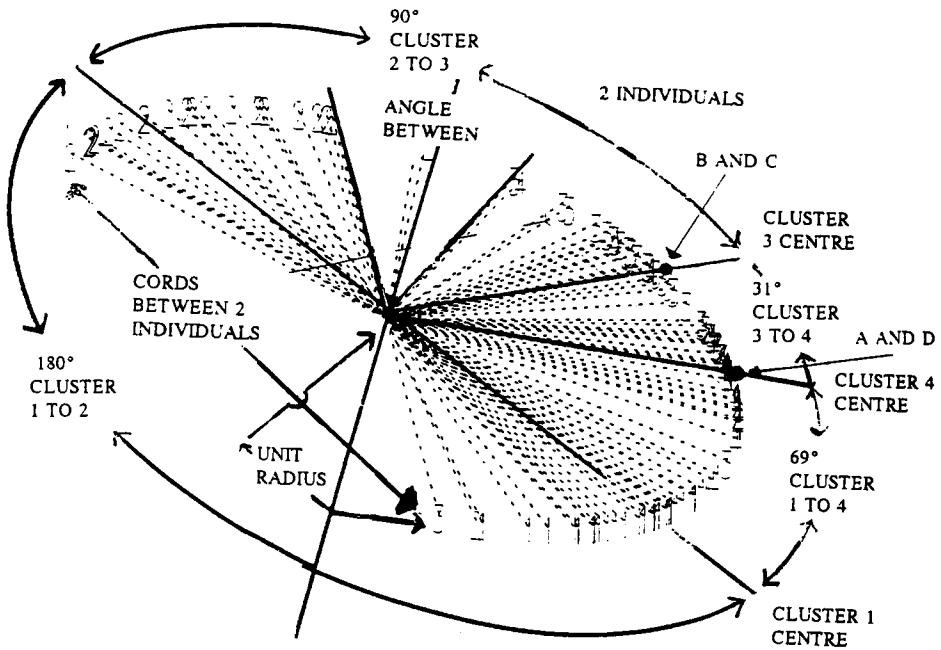
values can still result in many within cluster distances measures that are large. Because clusters are oblong, many points in a cluster can be closer to other clusters than to points within their cluster. Compared to Figure 4, points C and D from clusters 3 and 4 are relatively further apart but they are still closer to each other than to their respective cluster members B and A. Elongation as a result of ipsative amplitude must still be corrected.

A value for normalizing individual amplitude can be estimated from a person's range of deviation in variable values or by a computation such as the standard deviation in or vector length of the individual's variable values (e.g., $xstd(i,v) = xm(i,v) / amp(i)$). Both clustering and theoretical issues must be considered (Greenleaf, 1992, p. 179; Romesburg, 1990, pp. 84-85, Equations 7.5-7.8)² for this and related standardizations. When amplitude is defined by $(length = \sum x(i,v)^2)^{1/2}$ which is proportional to standard deviation, the object standardized variables are vectors of unit length. Such vectors all end on the surface of a unit hypersphere which is shown as a circle in Figure 6. In the case of 3-dimensional data, subtracting out the mean caused the loss of 1-dimension so the $Xstd(i,\cdot) = (xstd(i,1), xstd(i,2), xstd(i,3))$ vectors all end on a unit circle, a sphere in two dimensions. The numbers 1 through 4 on the circle show the cluster membership with ipsative factors and possibly ipsative information removed (Greenleaf, 1992). We are in a space in which distance should be measured. The clusters are clearly recognizable but there is overlap. The points A and D and the points B and C fall on each other rather than showing an inappropriate pattern of separation for the clusters as they do in Figures 3, 4, and 5. One way of finding clusters involves locating areas of high density (Figure 6). To clearly separate clusters 3 and 4 in Figure 6, however, one needs to also consider ipsative factors.

Selection of a Good Measure of Resemblance

Using $xstd(i,j)$ values in Equation 3 to find similarities or dissimilarities involves computing Pearson's r for objects. This occurs because in object standardization about a mean of zero, $D(i,j) = (\sum (xstd(i,k) - xstd(j,k))^2)^{1/2}$, where the k refers to variables. The formula defines the distance between the tips of 2 unit vectors that determine an angle so it gives the length of the cord of the angle. However, squaring and summing, $D(i,j) = (\sum xstd(i,v)^2 - 2(xstd(i,v)xstd(j,v) + xstd(j,v)^2)^{1/2}$. But since $xstd(k,v)^2$ for $k = i$ or j is 1 by the definition of $xstd$, and $\sum xstd(i,v)xstd(j,v) = r_p$, $D(i,j) = (2 - 2 r_p)^{1/2} = 2(1 - r_p)^{1/2}$. Deductively, one now sees that r_p , the correlation between two individuals' responses, provides the basis for a better approach for comparing two patterns conforming to the model defined by Equation 1 than Euclidean distance (Stanley & Beaman, 1993). Euclidean distance based on raw responses to attitude scales includes the ipsative mean

²It should be noted that Romesburg reverses rows and columns compared to how most SAS and SPSS users consider them. Columns are objects (e.g., people), and rows are variables.



The points A, B, C, and D correspond to the points in Figure 3.

Figure 6 Data transformations to visualize the r_p coefficient

and amplitude differences, resulting in a biased measure of distance (Cattell, 1949). The use of r_p as a resemblance measure is not new or unique (Cattell, 1949; Cohen, 1969; Holley & Guilford, 1964), but its application to recreation research has not been found. The fact derived here is that a model of behavior defined by Equation 1, requires r_p as a basis for a valid distance measure.

Since a valid distance measure should be a positive monotone strictly increasing function, the values of r_p , per se, do not define a distance function. However, $1 - r_p$, $(1 - r_p)^{1/2}$, and $1 - r_p^2$ have been suggested as distance measures (SAS, 1988). Three related distance measures based on r_p are defined in Equation 4:

- Equation 4A $D(i,j) = 1 - r_p$ The Correlation Distance Measure (e.g., Holley & Guilford, 1964)
- Equation 4B $D(i,j) = \arccos(r_p)$ The Angular Distance Measure or Arc
- Equation 4C $D(i,j) = (\sum(xstd(i,v) - xstd(j,v))^2)^{1/2} = (2(1 - r_p))^{1/2}$ Cord of Angle Distance

Figure 6 depicts the correlation measure in relation to angular distance between two people and the cord of the angle distance. Because the simulation included only 3 variables, all people are found on a circle. With 4 variables, all people would be on a sphere. Regardless of the number of variables, however, comparisons between two people will result in a circle defined by the two individuals' pattern vectors. Two different points on a hypersphere define a unique great circle route (i.e., the shortest distance). The angle and cord measures of distance should be thought of in relation to this circle.

Some examples may clarify the relationship of the three distance measures. If two individual's correlation is 1, the two people share a common pattern vector with a zero degree angle to each other. When the data for i and j are identical, $r = 1$, and $1 - r$, the dissimilarity or distance is zero (Equation 4A). Using Equation 4B, the angle between i and j is also zero since $\arccos(1) = 0$. Equation 4C gives the length of the cord of the angle distance which is also zero. If person i is in cluster 1 and person j is in cluster 2, their patterns are opposite and, if there is no random error, $r = -1$. The arccos of -1 is 180° . The cord of a 180° angle is simply the diameter of the unit circle so distance as defined by Equation 4A is 2, which is also the distance defined by Equation 4B. When $r = 0$, the arccos $(0) = 90^\circ$ (Equation 4B). The correlation distance is $1 - 0 = 1$ (Equation 4A). The cord distance is $2^{1/2} = \text{square root of } (1^2 + 1^2)$, since the cord is the hypotenuse of a triangle formed by two radii of length 1 that are at right angles.

The three resemblance measures in Equation 4 have not been defined to range from zero to 1 (or to some other common maximum), because these coefficients do not move from 0 to their maximum in the same way. Figure 7 shows how these and other related curves change with angle. Because the length of the cord of a unit circle is approximately equal to the angle in radians for "small angles," two of the three distance measures are very similar. At 90° , the angle in radians is 1.57, while the arc is 1.41; at 60° the angle is 1.05 while the arc is 1.00.

Though the absolute numeric value of a distance measure is not important, the merit of one function as opposed to another is in the effectiveness of recognizing which cluster a person is in. For Equation 4A ($D(i,j) = 1 - r_p$), distance changes slowly as the angle increases, then changes rapidly (Figure 7). In general, $1 - r_p$ represents a whole class of S-shaped functions in which distance increases slowly, then for a range of angles (e.g., 60° to 120°) increases rapidly (See curves A, D, & E). Above 120° , $1 - r_p$ again increases slowly. This does not have the same appeal as the two alternative measures, since an appropriate distance measure should increase uniformly (have a constant or monotone first derivative); not increase slowly, then increase quickly, then increase slowly as one nears 180° .

Of the other two transformations of r_p , the meaning of the value of angle of separation between vectors is more easily visualized (Equation 4B), than the length of the cord (Equation 4C). On the other hand, Equations 4B and 4C give virtually identical distances up to 90° . Although the linear relation between angle and cord breaks down to some extent above 90° , this

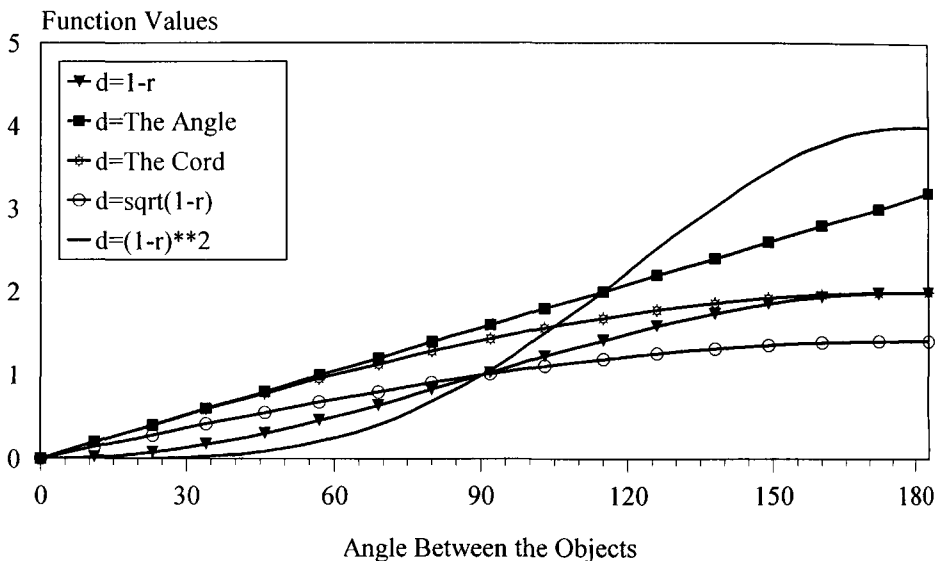
Functions of Pearson's r Between Objects

Figure 7 Five alternative distance functions

relation is less problematic "theoretically" than a S-shaped relation $(1 - r_p)$. One should note from Equations 4A and 4C that one has gone from the S-shape of $1 - r_p$ to a uniform increase by taking the square root of $1 - r_p$. In many clustering programs, distances based on Equation 4C would be squared, meaning that clustering criteria *would really* be based on an S-shaped curve of distance squared (see Figure 7). Practically, if points are separated by 90° and in the same cluster, the difference in cord, $1 - r_p$ and angular distance is simply not expected to significantly influence correctly associating points with their appropriate clusters.

In summary, a resemblance measure is not universal but rather a defined concept. Its merit lies in being good for the job intended. For our ipsative model (Equation 1), a transformation of r_p , is the basis of a valid distance coefficient to use. Of the three alternatives in Equation 4, $1 - r_p$, is rejected as "best" because it does not increase uniformly. The angle by definition increases uniformly *with angle*. This increase has intuitive appeal. From an interpretation perspective, angle can be thought of as a distance increasing just as distance between two objects increases on the surface of the earth. The cord as a distance measure does not have the same intuitive appeal: (1) one does not measure through the earth and (2) 90° is easier to understand than 1.41. The two measures, however, will give virtually identical results. In either case, the square of any angle or cord functions has appeal since one can reasonably believe that as angles get larger, distance should increase

more rapidly and increase one's confidence that the points are very dissimilar. However, when the cord distance is squared, one gets a measure proportional to that defined by Equation 4A (i.e., the S-shape). Therefore, although Equations 4B and 4C are similar, we recommend the angular measure $[\arccos(r_p)]$ or its square $[\arccos(r_p)]^2$. It increases "uniformly" up to 180° and is easier to interpret than the cord (Forgy, 1965).

Finally, it is important to note that although this paper has concentrated on Equation 1 and its relation to r_p , there are some situations where alternative attitudinal models and resemblance measures should be considered. For example, in importance-performance analyses, attitudes about performance on service delivery (e.g., clean washrooms, safety) can be viewed as a scale with an absolute zero. If the washrooms are not clean or there are no provisions for safety, performance is zero. In these situations, a model of performance ratings for individual, i , in aggregate, A , for a variable, v , can be defined as:

$$\text{Equation 5: } X_{A,i,v} = (C_A + c_{A,i}) (P_{A,v} + e_{A,i,v})$$

where the notation is based on Equation 1. However, $\Sigma P_{A,v} \neq 0$ and $P_{A,v} \geq 0$ for all v .

Equation 5 indicates that there are wide and narrow swing respondents based on $C_{A,i} > 0$ or $C_{A,i} < 0$. In addition, individuals show variation about their aggregate's P_A as indicated by $e_{A,i,v}$. There is no ipsative mean term, however, since there is an absolute zero point; a situation similar to forcing a regression line through the origin. In this instance, computing an individual's ipsative mean is not appropriate, since the mean will have statistical error associated with it. Subtracting out the mean introduces error and results in less accurate findings than there could have been. Thus, even though Equations 1 and 5 are closely related, Equation 5 defines distance as a function of the angle between two raw data vectors.

The appropriate distance measure for Equation 5 is the "cosine" measure, which compares two vectors, objects, independent of scale. This resemblance measure is appropriate for attitude data similar to performance scores that have an absolute zero (e.g., no performance on a given service). One could argue that importance also has an absolute zero. Whether a respondent is disposed to think of importance or performance as floating or anchored to some absolute zero depends on the context, and how the questions and response sets are structured. When performance has an absolute zero for each cluster, vectors of average performance emanate from the origin and go to the cluster center. Elongated scatterings of points occur in the general direction of these vectors, not perpendicular to a mean vector as shown in Figure 2.

Discussion

Based on simulation of individual differences in mean level and differences in pattern amplitude, the graphics presented here illustrate how dis-

tance measures are affected and why clusters are confused with each other. The dual ipsative transformation, a form of object standardization, eliminates this confusion for attitude data conforming to Equation 1. Although our findings are illustrated using only three variables, similar results have been produced by Grenier and Beaman (1993) using 24 variables and 5 clusters. Based on Equation 1, adding more variables does not result in less ipsative variability compared to pattern variability. Every variable has the ipsative mean and amplitude factors. Assuming Equation 1 is valid, the more variables included in the analysis, the more accurately the ipsative factors can be estimated and thus effectively separated from the pattern.

In the Grenier and Beaman (1993) analysis, simulation was used to produce 27 situations of the three types of variability: (1) narrow to wide variability in individual means, (2) individual amplitude, and (3) variable-by-variable random error. Clusters ranging from clearly separated groupings to clusters with no apparent structure were produced. For Grenier and Beaman (1993), traditional distance measures produced poor results compared to distance based on Equation 4A. As more and more of the three types of variability were introduced, only r_p approaches gave distances that resulted in most people being classed in their appropriate cluster. In other words, distances based on r_p defined correct clusters after the other methods failed. Because $1 - r_p$ worked when Euclidean distance did not, it was concluded to be a good measure to use when in doubt about whether there are ipsative effects.

Stanley and Beaman (1993) used both raw score variable standardized Euclidean distance clustering and Equation 4B, angular distance, for clustering data from an importance-performance study. Using Euclidean clustering, 5 clusters were found with 206 of 465 observations *not* in clusters. Three of the 5 clusters had under 30 members; one cluster had only 13 (a minimum cluster size of 12 was specified). Using the arccos of r_p distance, only 107 observations were *not* in clusters and larger clusters were found. Points placed in a cluster by one distance were not generally found in a particular cluster determined using another distance. From our results (Figures 4 and 5) and those of Stanley and Beaman (1993), we believe that Euclidean clustering based on variable standardized raw responses confuses spherical areas of apparent "high density" in the data with clusters. For Stanley and Beaman, "high density" in the raw data was really the ends of distorted circles coming close to each other near the mean vector (as illustrated in Figure 5 here). The reason for small clusters is that the far ends of distorted circles were "recognized" as clusters after certain areas near the mean vector were confused with clusters.

Although resemblance measures based on r_p have been reported in the literature for 50 years, their specific relation to the model presented here and their application in leisure research is not evident. In part, the application of Equation 1 and r_p has been limited because the most common statistical packages such as SPSS and SAS have not supported computing r_p

as an option for input data to cluster analysis.³ With the release of SPSS for Windows, however, r_p is available. One can use the techniques described here by using a clustering procedure that accepts distances based on object standardized input or that allows input of a resemblance matrix. Both SPSS and SAS offer such options. To use these latter approaches, however, the data must be "pre-processed" to create object standardized data or distances in one of the acceptable forms for input. This is obviously an impediment to use of r_p based measures.

This paper raises several analytical considerations. First, the data should be object standardized (not variable standardized). Second, a clustering procedure should not automatically perform a variable standardization on the object standardized data. In SAS and SPSS for Windows, for example, one can suppress variable standardization in input data for hierarchical clustering. When Euclidean distance is computed directly from object standardized data one gets distances based on Equation 4C (the cord); one of the r_p based distances. Third, if one needs a procedure that will handle a large number of cases effectively. FASTCLUS (SAS) or Quick Cluster (SPSS) can be used once the data are object standardized.

There are three cautions to consider when using r_p based distances implicitly or explicitly. First, since information can be lost when computations are performed, one must consider keeping and using this information (Greenleaf, 1992). When clusters have the same or very similar P_A vectors but different cluster general means, Equation 1 is a valid model. For clusters based on r_p , however, one must definitely examine individual means to distinguish clusters with similar P_A (Cattell, 1949). As discussed in relation to Figure 1, the curves 1 and 2 could theoretically identify different clusters than curves 3 and 4. One can visualize how clusters can collapse on each other by tracing what happens in Figures 4 to 6 with clusters 3 and 4 as ipsative corrections are applied. If these clusters had been directly above each other with different means, they would appear as a single cluster in both Figures 5 and 6. Using r_p based measures causes clusters with similar patterns but different means to collapse onto each other so that observations with a similar pattern are literally put into one cluster. Examination of IM_i (individual means), however, can be used to separate the observations into their appropriate clusters. Cluster estimation problems should routinely involve such an examination.

Second, amplitudes present a potential technical problem. In defining an amplitude one makes an estimate. Mathematically, the length of the vector is convenient for measuring amplitude and yields the "clean" unit hyperspheres seen as a circle in Figure 6. Unfortunately, for some distributions of observations, the standard deviation calculated in the usual way is highly

³Specialized cluster analysis programs that compute transformations of r_p directly from raw data have been available for years (See Romesburg, 1990, Appendix 2).

variable. If one has two unbiased estimates of amplitude, the first being highly variable compared to the second, the latter should be used. Greenleaf (1992) documents a variety of other issues related to amplitudes and information loss.

Finally, when the data conform to Equation 1, r_p should be used. If, however, one has data similar to performance data, Equation 5 is appropriate and r_p should not be used. Determination of which model and distance measure are most appropriate for a given data set may necessitate several analyses. Care must taken, however, to ensure that the data transformations do not give the impression that a method is superior simply because it transforms significant differences between observations into insignificant ones (e.g., as can occur when using log transformations in regression). A concern must be that reducing the elongation of collections of points more and more as points are further from a central value *artificially* creates concentrations of points that are then defined as "clusters."

By introducing formal models and arriving at an appropriate resemblance coefficient for a particular model (e.g., Equations 1, 2 or 5), analysts have criteria to use to reduce the risk of an inappropriate distance measure choice. We have not addressed, however, the matter of which cluster algorithm to use to actually find clusters from a distance matrix. Aldenderfer and Blashfield (1984) provide a good presentation of the theoretical and practical implications of using certain techniques. Following their advice, computing clusters using several clustering techniques and performing tests for cluster validity is a recommend strategy. Both the SAS and SPSS manuals provide examples that illustrate the advantages and disadvantages of different clustering techniques. Regardless of the clustering procedure, minimizing ipsative error before searching for clusters is an essential step in getting better results. Allowing ipsative factors to bias distance, distort the density measures, or otherwise mislead the cluster analysis is not appropriately compensated for by standard clustering algorithms.

A variety of other issues need to be addressed to advance the research presented here. First, the research showing that ipsative mean and amplitude carries information (Greenleaf, 1992) challenges the simplistic view that these ipsative factors can be ignored in *predicting* behavior and confirms concerns raised for half a century (Cattell, 1944). Multiple segment choice models offer a powerful potential tool for sophisticated analysis in this area (Bucklin & Gupta, 1992). Second, the issues raised by other researchers (Greenleaf, 1992; Grenier & Beaman, 1992; Stanley & Beaman, 1992) suggest that existing data should be re-analyzed to determine how the results change when using r_p or the "cosine." Third, recent research (Schwarz, et al., 1991; Gilljam & Granberg, 1993) addresses issues regarding distortions in scales, and the need for negative "anchors." Since distortion of distance patterns causes problems in determining cluster membership, acquiring new data sensitive to ipsative variations should be a priority.

Conclusions and Management Implications

The analysis model proposed here can be expected to be a "better" model than a variety of alternatives. Although the appropriate resemblance coefficient arrived at for the model defined by Equation 1 is not new, it is correct for that model. A variety of literature offers improvements to r_p and makes unqualified statements about the superiority of these alternatives. One can argue, however, that the "cosine" is the appropriate distance measure for attitudes like performance that have an absolute zero. Given no model, one can propose measures may have technical merit (e.g., the use of non-parametric statistics when the distribution is unknown), but this is a separate issue. In cases where the model defined by Equation 1 is a good fit to reality, the r_p method of measuring distance is appropriate. If the data conform to Equation 5, the "cosine" measure is appropriate.

It is important to recognize that analysis based on r_p can produce valid results when Euclidean and other resemblance coefficients do not detect a structure (Aldenderfer & Blashfield, 1984, p. 59). More importantly, clustering with an inappropriate resemblance coefficient (e.g., Euclidean) can actually detect an invalid structure when data conform to Equation 1. However, r_p is valid for data that conform to both Equations 1 and 2, subject to examination of IM_i . This means that where Equations 1 or 2 may apply, using the r_p based distance approach offers valid estimates regardless of whether the Equation 1 model is really needed. Even if data are expected to be approximated by Equation 2, we believe that one should use an r_p based distance when there is a pattern to detect and a chance that ipsative effects will distort Euclidean distance. On the other hand, if the data are best approximated by Equation 5, using r_p is statistically inefficient and may produce invalid clusters.

When Equation 1 applies, an r_p type measure minimizes ipsative bias in defining a spatial relationship among objects. Such reductions in bias increase the likelihood of finding valid clusters. Adopting the procedures described here can increase one's trust in the quality of the findings. From a research perspective, Equation 1 will achieve efficiencies in the use of data. Because "extraneous" variance related to individuals is appropriately eliminated, one can collect less data than necessary if ipsative variance is treated as part of "residual error." Getting more accurate results with less data and having valid clusters to be used in analysis in making decisions are good reasons to use a function of r_p .

Getting valid clusters has ramifications for management. This work arose out of importance-performance analysis that was seen to be questionable because valid social aggregates were not recognized by Euclidean distance clustering (Stanley & Beaman, 1993). Starting analysis by denying the existence of competing user groups yields invalid results if such groups exist. Finding invalid clusters by using an inappropriate distance measure is a waste of resources. Failure to recognize social aggregates renders claims of client

orientation research meaningless. Given the suggested model, researchers using attitude scales have an approach to finding valid social aggregates for planners and managers to consider in relation to their proposed actions.

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